Math 2050, HW 5

Q1. Let $a \in \mathbb{R}$ and $f : [0, a)$ be a real valued function given by $f(x) = x^4$.

(a) Show that f is uniformly continuous.

(b) Is the conclusion in (a) still true if a is replaced by $+\infty$? Justify your answer.

Q2. Let $f : [0, 1] \to \mathbb{R}$ be a real valued function given by $f(x) = x^{1/3}$.

(a) Show that f is not a Lipschitz function.

(b) Using ε, δ terminology, show that f is uniformly continuous.

Q3. Let $f : [0, +\infty) \to \mathbb{R}$ be a continuous real valued function.

(a) Suppose there is $L, k > 0$ such that for all $x > k$, $|f(x)| \leq L$. Prove that f is uniformly bounded by showing that there exists $\tilde{L} > 0$ such that for all $x \in [0, +\infty)$, $|f(x)| \leq \tilde{L}$.

(b) Suppose $\lim_{x \to +\infty} f(x) = \alpha \in \mathbb{R}$, show that f is uniformly continuous on $[0, +\infty)$.

Q4. Let $f : [0, 1] \to \mathbb{R}$ be a real valued function such that f is continuous and $f(x) \notin \mathbb{Q}$ for all $x \in [0, 1]$. Show that f must be a constant function.